Project 1

Surveyor

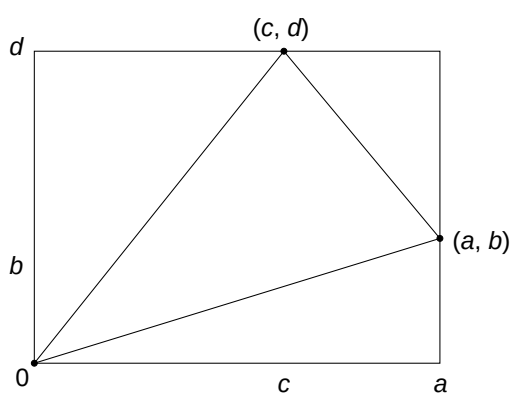
Introduction

When surveyors survey a tract of land, they locate a sequence of points around the perimeter of the land. The tract of land itself is the polygon determined by these points as vertices. In this project, we will write a program named `Surveyor.cpp` that will read the coordinates of exactly six points and then calculate the area of the tract of enclosed land. The program will output the area in both square feet and acres.

In practice, surveyors work with the distance and bearing from one point to the next. For example, when going from point 1 to point 2, they may say to go 16.55 feet on a bearing of N64°58'59"E. To greatly simplify our program, we will use the x and y coordinates instead. So we might say that point 1 is at $(3, 5)$ and point 2 is at $(10, 20)$. In other words, surveyors us polar coordinates, but we will use rectangular coordinates. The origin of our coordinate system is arbitrary. You may choose it to be at point 1, but it does not matter.

Finding the Area of a Polygon

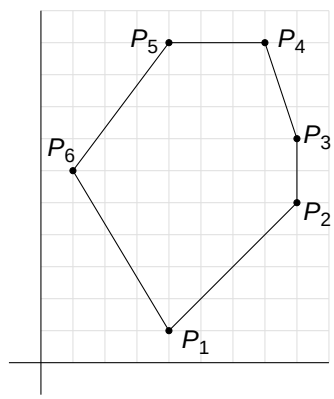
To find the area of a general polygon, let us first consider the simple triangle below, with one vertex at the origin. We may find the area of this triangle by enclosing it in a rectangle and then removing the area in the rectangle that is not part of the triangle.



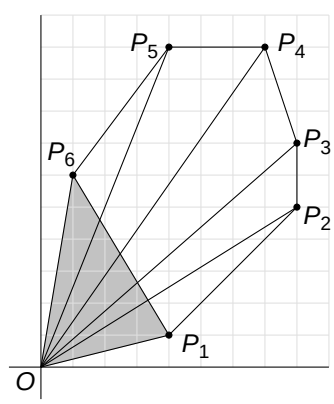
The rectangle has area ad; the lower right triangle has area $\frac{1}{2}ab$; the upper left triangle has area $\frac{1}{2}cd$; and the upper right triangle has area $\frac{1}{2}(a-c)(d-b)$. Therefore, the original triangle has area

$$\begin{aligned}\text{Area} &= ad - \left(\tfrac{1}{2}ab + \tfrac{1}{2}cd + \tfrac{1}{2}(a-c)(d-b)\right)\\ &= ad - \tfrac{1}{2}(ad+bc)\\ &= \tfrac{1}{2}(ad-bc).\end{aligned}$$

Now consider a polygon with vertices $P_1, P_2, \ldots, P_6$.



For each side P_iP_{i+1}, we may form a triangle OP_iP_{i+1}, where O is the origin (or any fixed point).



Then we use the above formula for the areas of these triangles. Note that in this example, the areas of triangles OP_1P_2, OP_2P_3, OP_3P_4, OP_4P_5, and OP_5P_6 will be positive (measured counterclockwise) and the (gray) area of triangle OP_6P_1 is negative (measured clockwise). Therefore, when we add the six areas together, we are left with the area of the polygon! That is, by subtracting the area of triangle OP_6P_1, we correct for the extra area included by the other 5 triangles. This always works.

In our example, the six points are (4, 1), (8, 5), (8, 7), (7, 10), (4, 10), and (1, 6). Therefore the area of the polygon is

$$\begin{aligned}
\text{Area} &= \tfrac{1}{2}(4 \cdot 5 - 1 \cdot 8) \\
&= \quad +\tfrac{1}{2}(8 \cdot 7 - 5 \cdot 8) \\
&= \quad +\tfrac{1}{2}(8 \cdot 10 - 7 \cdot 7) \\
&= \quad +\tfrac{1}{2}(7 \cdot 10 - 10 \cdot 4) \\
&= \quad +\tfrac{1}{2}(4 \cdot 6 - 10 \cdot 1) \\
&= \quad +\tfrac{1}{2}(1 \cdot 1 - 6 \cdot 4) \\
&= 6 + 8 + 15.5 + 15 + 7 - 11.5 \\
&= 40.
\end{aligned}$$

The Program

Write a program named `Surveyor.cpp` that will read in the coordinates of six points. For each of exactly six points, the program should prompt the user for the x and y coordinates of each point. Assume that the units are feet. The program will then calculate the area in square feet, using the above formula and print out the area in square feet and in acres. There are 5280 feet in a mile and 640 acres in a square mile. In our example, 40 square feet would convert to 0.000918 acres. The output should be properly annotated.

Turning in Your Work

Place your file `Surveyor.cpp` in a folder named `Project 1` and drag the folder to the dropbox. Be absolutely sure that the file name and folder name are correct. Otherwise, I might not find your work. Your work is due by Friday, September 20, 2019. Do not underestimate the time it will take to complete this project. Get an early start! It is ok to turn it in early.